Differential Equations – Separation of Variables

A separable differential equation is one which is in the form dy/dx = g(x) h(y). In other words it is an equation where your dy/dx is equal to a function containing the product or quotient of a function of x and a function of y. **To take the integral we will have to separate the variables and put the y's on one side with the dy and the x's on the other with the dx.** Then you take the antiderivative of both sides. The only trick is to be sure to follow the rules of integration and to be sure to use substitution whenever it is needed.

Now I will do an example here. There will be more examples in the homework examples.

A:

$\frac{dy}{dx} = \frac{e^x(y-1)}{-2(e^x+4)}$	This is the differential equation. We can't take the integral yet because we have x's and y's mixed together on the right side of the equation.
$\frac{dy}{y-1} = \frac{e^x}{-2(e^x+4)}dx$ $\int \frac{dy}{y-1} = \int \frac{e^x}{-2(e^x+4)}dx$	We will separate the variables by dividing by the y - 1 and multiplying by dx. Set up the integral.
On the right side: $u = e^x + 4$ $du = e^x\,dx$ $-\frac{1}{2}\int \frac{1}{u}du = -\frac{1}{2}\ln\|u\|$ $\ln\|y - 1\| = -\frac{1}{2}\ln(e^x + 4) + C$	Now we take the integrals. I have to use substitution on the right side of the equation. You can stop here! I will simplify some more but unless your directions say to find y = f (x), you do not have to solve for y.
$2\ln\|y - 1\| + \ln(e^x + 4) = C$	This can actually be simplified some more. I moved the right side to the left and multiplied by 2. 2C is the same as C since it is just some constant.
$\ln(y - 1)^2(e^x + 4) = C$	Use the laws of exponents to simplify.
$e^C = (y - 1)^2(e^x + 4)$ or $C = (y - 1)^2(e^x + 4)$	Now rewrite this in exponential form. e^C is just a constant that we can rename again.

I think that the trickiest part of this section will be to find all of the antiderivatives without making silly mistakes. The concept of separating the variables is probably not too difficult. So ... be careful before taking any integral and make sure you are following the proper procedures: substitution, etc.